Rational Dilemmas and their Place in the Bigger Picture

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1 Introduction

Normative dilemmas are a familiar feature of life. One is obligated to do two distinct things, though it is impossible do both. One way or the other, one is going to fail to do what one should. The potential for dilemmas is present in any system of norms. But, one might think, the norms of rationality ought to be free from such things. How can it ever be possible to end up in such a bind if one is rational? But one can.

In this essay I will discuss such matters and their bearing on a number of related topics. There is, perhaps, nothing very new in what I have to say, but what I hope to do is to lay out the place of rational dilemmas in a wider context, comprising normative dilemmas in general, dialetheism and paradox, and the logic of normative reasoning.

2 Preliminary Matters

Let us start with some preliminary matters. Norms are systems of rules, which determine what someone must, must not, may, or may not do. The words *must* and *may* are, however, highly ambiguous, and may express other kinds of modalities—e.g., physical, epistemic, logical.[1] The kind of modality we are concerned with here is normative. But normative modality can itself be of different kinds. A non-exhaustive list includes the following:

- *Game norms*. In chess, you may move a rook along a rank or file in any direction and as far as is unimpeded; you may not move it along a diagonal.

- *Club norms*. It is still the case that in some clubs women may not join, but that members must wear a tie to dine.

- *Legal norms*. In New York, you must drive on the right hand side of the road. You may turn right on a red light provided it is safe to do so.

- *Moral norms*. You may donate to a charity, but it is not the case that you must do so.

- *Rationality norms*. All the evidence points mercilessly in favour of such and such a conclusion, so you must believe it. Some evidence may support different views equally.

When *must* and *may* are used to express normative modality, they can be replaced by a variety of near synonyms, the precise choice being dependent on contextual factors concerning idiom and implicature. Thus, for *must*, one has, for example: *is obligated to*, *is obliged to*, *ought to*, *is required to*; and for *may*, one has *is allowed to*, *is permitted to*. The kind of deontic modality at issue may be clarified by using an adverb, as in: *it is* legally *required to*, *it is* morally *permitted to*.

Statements using such locutions, when the verb is active, have a subject. (See all the above examples.) And the subject (agent) may be highly relevant to the norm. Thus, I may not go into a female toilet, but my wife may. In what follows, variation of the agent will play no significant role, however, and I will just assume it fixed. When necessary I will use the subject *you*.

With a bit of regimentation, we can employ, not a modal verb, but an adverb which prefixes the whole sentence. Thus, instead of saying *you are permitted to turn right on a red light*, one may say *it is permitted that you turn right on a red light*. Such prefixes are what logicians call modal operators. *It*

[1] See, e.g., Priest (2008), 3.6.

must be the case is standardly written as $\Box$, and *it may be the case* as $\Diamond$. In what follows I will take these operators to express deontic modality. If one wishes, one may add a subscript to indicate the kind of norm in question. So $\Box_L$ might be legal obligation, $\Diamond_M$ might be moral permissibility, etc. However, in what follows, the point being made either does not depend on the kind of norm in question, or that kind is obvious from the context, so subscripts will be unnecessary.

3 Dilemmas and Dialetheias

Now let us turn to the subject of dilemmas. We have a dilemma when there is a pair of true statements of the form: *you must do x*, and *you must do y* (the *must* being the normative *must*), but where it is impossible to do both; that is, for whatever reason (conceptually, physically, or whatever) you are unable (cannot) realise both obligations. Let us write $\blacklozenge$ for the kind of possibility at issue here, and $\blacksquare$ for the corresponding kind of necessity. One may then read $\blacksquare(A \rightarrow B)$ as saying that you can't (in the relevant sense) bring it about that A without bringing it about that B; so $\blacksquare(A \rightarrow \neg B)$ means that whatever you do, if you bring about A, B won't be the case.[2] We may, then, take a dilemma to be of the form: $\Box A$, $\Box B$, and $\blacksquare(A \rightarrow \neg B)$.

Under certain assumptions we may take a dilemma simply to be of the form: $\Box A$ and $\Box\neg A$. That this expresses a dilemma follows if $\blacksquare(A \rightarrow \neg\neg A)$ holds. Now, $A \rightarrow \neg\neg A$ is, presumably, a logical truth. Hence $\blacksquare(A \rightarrow \neg\neg A)$. (You can't bring it about that A without bringing it about that $\neg\neg A$!)

In the other direction, we need the inference: $\blacksquare(A \rightarrow B), \Box A \models \Box B$. For then, $\blacksquare(A \rightarrow \neg A)$ and $\Box A$ give us $\Box\neg A$. One might contest this inference, but there clearly seems to be some sense in which, if you can't do x without doing y, and you are obliged to do x, you are obliged to do y. If you can't post the letter withought leaving home, and you are obliged to post the letter, you are obliged to leave home.

Note that a dilemma, even in the form $\Box A$ and $\Box\neg A$, is not a dialetheia. That is, it is not a pair of true statements of the form B and $\neg B$. What would be a dialetheia is a pair of true statements of the form $\Box A$ and $\neg\Box A$. If it were the case that $\Box\neg A \models \neg\Box A$ then a dilemma would entail a normative dialetheia. But such an entailment is implausible if there are dilemmas,

[2] And *vice versa*, given the contraposibility of $\rightarrow$.

simply because it *can* be the case that $\Box\neg A$ and $\Box A$.[3]

This raises the issue of what, exactly, the logic of $\Box$ is. I defer a discussion of the matter to an appendix, since the rest of this essay does not depend on it.

4 Legal Norms

Let us now turn to matters concerning legal norms.[4] It is clear that there are legal dilemmas. Thus, this can happen, for example, in contract law. Suppose that you contract to, say, sell your house to person, a, under circumstances X, and contract to sell your house to a different person, b, under circumstances Y. You may take it, mistakenly, that circumstances X and Y cannot both arise; or that their joint arising is so improbable as not to be worth considering; or it may not even occur to you that this is a possibility. (Contracts are often complex and opaquely worded.) But suppose that circumstances X and Y do, in fact, arise. Then you are legally obliged to sell the house to a, and legally obliged to sell the house to b, though you cannot do both. The fact that you have contracted to different parties does not relieve you of the obligation to each, as is shown by the fact that you may be sued by whichever party you do not, in fact, sell the house to. So we have a dilemma concerning legal obligation.

As I have already noted, legal dialetheias are a different matter again, though they are certainly possible as well. Suppose one law says that:

- all persons in category X may vote

and another law says that:

- all persons in category Y may not vote

One might suppose that at the time of legislation there was no serious possibility that someone might be in both categories, X and Y. (Thus, X might be *being black*, or *being a woman*, and Y might be *owning property*.) And as long as no one *is* in both categories, matters are consistent. But if and when

[3]Of course, if dialetheism is on the table, the inference could be valid, and every dilemma would deliver a dialetheia of the form $\Box A$ and $\neg\Box A$. But this would seem to multiply contradictions beyond necessity, and so is to be avoided. See Priest (2006a), 8.4.

[4]This section draws on Priest (2006a), 13.1 and 13.2.

someone who is in both categories does appear, that person both may and may not vote.

Of course, the law has a way of defusing some contradictions of this kind. Thus, in most jurisdictions, laws are ranked in kind, in increasing order of strength: case law, statute law, constitution law; and a higher ranked kind of law takes precedence over a lower ranked kind of law. Or again, in many jurisdictions, there is a principle of *lex posterior*, according to which, in cases of conflict, a later law takes precedence over an earlier law. But it is easy to envisage situations where no such principle applies: for example, in which the two laws were part of the very same piece of legislation. Of course, if such a situation were to arise, the legislation would be amended to make it consistent, or a judge would make an appropriate ruling (which would amount to the same thing). But it remains the case that, before the change, both clauses of the law were operative—indeed, that was exactly why a change was required. Thus, for example, the second clause might be appealed to to establish that a certain poor black person has no right to vote. So before the change, we have a legal dialetheia.

I note that normative dialetheias may arise, not only with the law, but with the conceptually similar rules of clubs. One hypothetical example, clearly modelled on Russell's paradox, is given by Chihara.[5] A certain group of people decide to form a club, and they have its consitution duly ratified. There is one simple condition for membership. Membership shall be open to all and only those who are members of clubs which they are not eligible to join. After a while, the club expands, and they appoint a secretary, who then applies for membership. Is the secretary eligible to join? They either are or aren't. But if they are, they aren't; and if they aren't, they are. So, it would seem, they both are and aren't. None of the legal escape mechanisms is available in this case. And until the club's rules are changed, the applicant clearly has contradictory normative properties.

5 Moral Norms

Let us now move to the case of moral norms. That there are moral dilemmas may be established in the same way that we established that there are legal

[5]Chihara (1979). For reasons that have always been unclear to me, he does not draw the obvious dialetheic conclusion.

dilemmas. We simply appeal to promising instead of contracting.[6]

Thus, suppose that you promise in good faith to meet person a at a time and place of their choosing. And you promise in good faith to meet person b at a time and place of their choosing. a and b then choose different places, but the same time. You may have thought it would be impossible that this situation could arise; or thought it so unlikely as not to be worthy of consideration; or you may even have made one of the promises forgetting about the other. It remains the case that you are morally obliged to be in both places at the same time, which you cannot be.

Now, some apparent moral dilemmas are only *prima facie*. You promise to meet someone for a drink, but are unable to make it because an unforeseen circumstance arose in which you were obliged to stop to save someone's life. Clearly, the second obligation takes precedence over the first. Or you make promises that turn out to be incompatible, but someone's life depends on one, but nothing much depends on the other. Again, one may reasonably take the more important promise to takes precedence over the other.

But it is entirely possible that there is nothing to break a tie in promises. Both people to whom you made the promise are equally important to you. (Maybe they are your two children.) Keeping the promise is equally important for both of them. (Maybe you were going to give each child their birth certificate, which each needs for the same purpose.) Maybe you even made the two promises at the same time (say, in the same email). Neither promisee is willing to "let you off" your promise. And so on. Matters are entirely symmetrical; and since they are symmetrical, you are obliged to keep both promises or neither. But making a conflicting promise does not remove an obligation to keep a promise. That would make it all too easy to break a promise. So both promises stand, and you have a dilemma.[7]

Establishing that there are moral dialetheias is harder than establishing that there are legal dialetheias. It is easy enough to find moral principles that contradict each other, for example: *thou shall not kill* and *one may kill in self defence*. And doubtless there are conditional principles that produce

[6] For a general discussion of moral dilemmas, see McConnell (2018).

[7] Perhaps one of the most famous moral dilemmas is *Sophie's choice*. Sophie is forced by a concentration camp guard to save one of her children by condemning the other. I suppose that, just possibly, one might hold her to have merely a disjunctive obligation in this situation: she has an obligation to save either one child or the other. But even this cannot be the case with the promising example of the text, since making an incompatible promise does not, *ipso facto*, relieve you of an obligation imposed by the other.

contradiction when applied in contingent circumstances, such as, things of the form: if you are in category X, you are morally permitted to do such and such; if you are in category Y, you are not permitted to do such and such. The trouble is that such principles, categorical or conditional, normally come with (usually somewhat vague) *ceteris paribus* clauses. In other words, they provide only *prima facie musts* and *mays*. It is usually all too easy to suggest that when properly articulated these principles do not conflict. Indeed, the fact that they would otherwise deliver a contradiction provides an obvious basis for doing so.

The same is true of promising, of course. If you make a promise you have an obligation to keep it. But as I have already observed, the obligation is only *prima facie,* since it may be outweighed by more important considerations. What makes it possible to use this *prima facie* principle to deliver the moral dilemma above is the symmetry of the situation. Since the promises and the circumstances surrounding them are symmetrical, there is nothing that could affect one obligation without affecting the other. So either both promises are operative or neither is. And simply having made an incompatible promise of equal weight does not appear to be a good ground for voiding the obligation of a promise, as I observed.

The reason it was much easier to establish the existence of legal dialetheias than of moral dialetheias is that it is we who make laws, and they are spelled out in black and white in the appropriate legislation. Of course, there are always matters of interpretation, but this is a quite different situation from having no agreed upon statement at all.

The analogy with legal dialetheias certainly suggests that there are moral dialetheias; but I know of no detailed examples which I find compelling. In any case, the main topic of this paper is rational norms. So let us pass on to these.

6 Rational Dilemmas 1: Rational Action

It might be thought that there is something special about norms of rationality such that they could not produce dilemmas. As we will now see, this is false. A first kind of example appeals to a certain principle of rational choice. Suppose that you have to choose between two actions, Action$_1$ and Action$_2$. The only relevant concern is the result which each delivers. Action$_1$ delivers Outcome$_1$; Action$_2$ delivers Outcome$_2$. Outcome$_1$ is better than

Outcome$_2$. In that case then, at least if all this is known to you, you should choose Action$_1$. That is, $\Box$(you choose Action$_1$), where the box expresses the rational *must*. Let us call this principle RC.

Applying RC in certain symmetric situations delivers a dilemma. Here is one, which is a symmetrised version of the Prisoners Dilemma.[8] We suppose that you are in a room, and you have to choose between pressing button a and button b. You cannot press both. (Pressing one, we may suppose, disables the other. And if you press neither, we may also suppose, you will be shot.) There is a slot in the wall which can deliver money to you. (And we measure how good an outcome is by the amount of money you receive.) In the next room, there is someone who is in exactly the same situation as you. If you press button a then, in virtue of that, you will receive \$10 and the other person will receive \$0. If you press button b then, in virtue of that, you will receive \$0, but the person in the next room will receive \$100. All this is known to you. And here's the rub. You cannot communicate with the other person, but you have known them for a long time, and you know that they think exactly like you in matters such as this. (Anyone who has been married for a long time can have exactly this kind of knowledge about their partner.) We now argue as follows.

The person in the other room will press either button a or button b, and you have no control over what this is. Let x be the amount of money you will receive in virtue of their action. Then if you press button a you will receive \$$(10+x)$. If you press button b you will receive \$$x$. $10 + x > x$, so by RC, you should press button a.

On the other hand, if you press button a so will the other person, so you will (both) receive \$10. But if you press button b, so will the other person, so you will (both) receive \$100. Since 100>10, applying RC, you should press button b.

Since RC and the logic of these arguments are uncontentious, the only hope for breaking the conclusion requires one to reject one of the premises. The only real hope for this, it seems to me, concerns the first argument, and is to say that 'x' should be considered as a definite description, which changes its value in the two conditionals stated—giving rise to a fallacy of ambiguity. But 'x' does not have to be understood in this way. We may simply take it to be a rigid designator for whatever amount of money it is that is delivered

[8] What follows draws on Priest (2006b), 6.7. There I give another example of the same kind, which is a version of Newcombe's Paradox.

by the other person's action.

7 Rational Dilemmas 2: Rational Belief

Rational dilemmas of a quite different kind can be produced with some inspiration from the paradoxes of self-reference—just as Chihara's rule dialetheia was inspired by Russell's paradox.[9] In particular, one can produce such a dilemma by appealing to the thought that it is irrational to both believe something and, at the same time, believe that it is irrational to believe it. This seems highly plausible. If someone believes A, and, at the same time, believes that it is irrational to believe A, that would seem to be itself pretty irrational.

Call this principle IB. Now consider the claim: it is irrational to believe this very claim. Call that claim C. Suppose that one believes C. Then one believes C and at the same time believes that it is irrational to believe C. This is irrational. Hence one ought not to believe C. But this is exactly what it says. So we have demonstrated C. Assuming that if something has been shown to you to be true, you should believe it, you ought to believe C.

The reasoning here is a bit more slippery than in the decision-theoretic example of the previous section, so let me formalise it. Let $\mathfrak{B}x$ be the predicate 'you believe x'. Then IB is the following schema:

- $\Box\neg(\mathfrak{B}\langle A\rangle \wedge \mathfrak{B}\langle\Box\neg\mathfrak{B}\langle A\rangle\rangle)$

where $\Box$ is the *must* of rationality, and angle brackets indicate a name-forming functor. By standard techniques of self-reference, we can produce a sentence, ρ, of the form $\Box\neg\mathfrak{B}\rho$. Applying IB to this, we get:

- $\Box\neg(\mathfrak{B}\rho \wedge \mathfrak{B}\langle\Box\neg\mathfrak{B}\rho\rangle)$

That is:

- $\Box\neg(\mathfrak{B}\rho \wedge \mathfrak{B}\rho)$

which is logically equivalent to:

- $\Box\neg\mathfrak{B}\rho$

[9] The following draws on Priest (2006b), 6.6.

But this is ρ, and we have just established it. If something is established, one should believe it. Hence, we have:

- $\Box\mathfrak{B}\rho$

The last two lines are a dilemma. The only real possibility for rejecting the argument would seem to be to reject IB itself; and that would appear to be an act of desperation. Just possibly, a dialetheist might hold that it is not irrational to believe something and, at the same time, believe that it is irrational to believe it, if they believe that it is *also* rational to believe it, though even this is moot.

8 Rational Dialetheias

With a minor tweak, the argument of the last section delivers not just a rational dilemma, but a rational dialetheia. If we slip the negations in IB out of their modal scopes, we get IB^*:

- $\neg\Box(\mathfrak{B}\langle A\rangle \wedge \mathfrak{B}\langle\neg\Box\mathfrak{B}\langle A\rangle\rangle)$

which seems equally cogent, if not more so. A rational person may have no beliefs about rationality itself. They may not even possess the concept. So it cannot be rationally necessary to believe the conjunction of two things, one of which is a statement about rationality.

We can now run the argument essentially as before. This time, let ρ be of the form $\neg\Box\mathfrak{B}\rho$. Applying IB^* to this, we get:

- $\neg\Box(\mathfrak{B}\rho \wedge \mathfrak{B}\langle\neg\Box\mathfrak{B}\rho\rangle)$

That is:

- $\neg\Box(\mathfrak{B}\rho \wedge \mathfrak{B}\rho)$

which is logically equivalent to:

- $\neg\Box\mathfrak{B}\rho$

But this is ρ, and we have just established it. If something is established, one should believe it, Hence, we have:

- $\Box\mathfrak{B}\rho$

We then have a dialetheia.

Of course, one might contest this reasoning. There is not much to contest in the validity of the argument, so if there is going to be a problem, it has to be with IB^*. And that doesn't look terribly promising, as noted.

This is obviously some kind of paradox of self-reference. Standard such paradoxes deploy either set-theoretic or semantic notions. This paradox is distinctive since it contains none of these, but contains normative notions instead. As probably hardly needs to be said, solutions to the standard paradoxes of self-reference are legion.[10] Perhaps the most commonly touted solution for the semantic paradoxes is some version of rejecting the Principle of Excluded Middle, $A \vee \neg A$. As a moment's thought shows, though, this principle is not appealed to in the above argument. Assuming, as seems reasonable, that this is a paradox of the same kind as the other paradoxes of self-reference, and assuming that paradoxes of the same kind should have the same kind of solution—the Principle of Uniform Solution[11]—this shows that the standard paradoxes are not to be solved by rejecting Excluded Middle.

9 Conclusion

So much for normative dilemmas and dialetheias. We have had a closer look at some of these; but I have also located them in the context of normative dilemmas and dialetheias in general. Standard paradoxes of self-reference have surfaced a couple of times, but perhaps the most striking thing about the foregoing discussion is the frequency with which considerations of symmetry have appeared. Since normative dilemmas and dialetheias have two symmetric parts, reflected round the axis of a negation, perhaps this is not surprising. At any rate, I hope that putting normative dilemmas and dialetheias in this more general context has helped to put them in a new focus.[12]

[10]See, e.g., Bolander (2017).

[11]See Priest (1995), 11.5.

[12]Many thanks go to Scott Stapleford and Kevin McCain for their very helpful comments on a first draft of this paper.

10 Appendix: the Logic of Normative Dilemmas and Dialetheias

In this appendix I will discuss the formal logic of normative dialetheias and dilemmas.[13] To keep matters simple, I will assume for the moment that dilemmas are of the form $\Box A$ and $\Box\neg A$. Normative dialetheias are, of course, of the form $\Box A$ and $\neg\Box A$. I will briefly discuss $\blacksquare$ and $\blacklozenge$ at the end of the appendix.

The usual semantics for modal operators is a world-semantics.[14] This is based on classical logic, and the truth conditions of the modal operators are given in terms of a binary accessibility relation, R. (Intuitively, in the case of normative modality, xRy means that all the obligations that hold at x are realised at y.) So $\Box A$ is true at a world w just if for every world, w', such that wRw', A is true at w'; and $\Diamond A$ is true at a world w just if for some world, w', such that wRw', A is true at w'. $\Diamond A$ may be defined as $\neg\Box\neg A$, or $\Box A$ may be defined as $\neg\Diamond\neg A$.

In the simplest such logic, K, there are no constraints on the relation R. Stronger systems are obtained by imposing such constraints. Some standard ones worth noting in the present context are as follows:

- Reflexivity, $\forall x\, xRx$. This gives the inference $\Box A \models A$. One certainly does not want this in the case of deontic modality, since—notoriously—people do not always do what they ought, and sometimes do what they ought not.

- Seriality, $\forall x \exists y\, xRy$. This is standard in deontic logics. It gives the inference $\Box\neg A \models \neg\Box A$. As already noted, one should not have this if there are dilemmas; for then, dilemmas collapse into contradiction.

- Symmetry, $\forall x \forall y (xRy \rightarrow yRx)$. This delivers the inference $A \models \Box\Diamond A$. One certainly does not want this in a deontic logic. The fact that A is true hardly makes it deontically permitted, let alone permitted in an obligatory fashion.

- Transitivity, $\forall x \forall y \forall x ((xRy \wedge yRz) \rightarrow xRz)$. This delivers the inference $\Box A \models \Box\Box A$. This is more plausible, but it is still moot. Suppose that

[13] For somewhat different approaches, especially concerning the conditional, see Priest (2006a), ch. 13 and §19.15, and Routley and Plumwood (1989).

[14] See Priest (2008), chs. 2, 3.

you murder someone. Then you ought to be punished, but you ought not to have murdered the person in the first place, so it oughtn't to be the case that you ought to be punished.

- Euclideanness, $\forall x \forall y \forall z((xRy \wedge xRz) \rightarrow yRz)$. This delivers the inference $\Diamond A \models \Box\Diamond A$. This is similarly problematic. Suppose someone hurts you. Then it is certainly permissible to forgive them. But since they ought not to have hurt you in the first place, it oughtn't to be the case that you may forgive them.

But even without any further constraints on R, if there are dilemmas, these semantics are inadequate. True, if x is a world such that for no y, xRy, then $\Box A \wedge \Box\neg A$ is true at x. The problem is that in these semantics $\Box A \wedge \Box\neg A \models \Box B$. So if a dilemma arises, everything becomes reqired. That's crazy. If you find yourself in a moral dilemma because of promising, it hardly follows that you ought to murder the promisee. (And if there are normative dialetheias, matters are even worse, since $\Box A \wedge \neg\Box A \models B$ (*Explosion*). So if such a dialetheia arises, everything is true—even things that have nothing to do with norms—e.g., that you are a fried egg.)

That one cannot use the semantics of classical logic if there are normative dialetheias is obvious, since these validate Explosion; but it is just as true if there are normative dilemmas. If there is such a dilemma, $\Box A \wedge \Box\neg A$, at world, w, and not everything is normatively required, there must be some accessible world, w', where both A and $\neg A$ are true at w'. This is not posible in the semantics of classical logic. A semantics which allows us to accommodate both dilemmas and dialetheias is that of a paraconsistent logic. Let me illustrate with the simple paraconsistent logic LP.[15] The language of this contains only the connectives $\neg$, $\vee$, and $\wedge$.

An interpretation is a structure, $\langle W, @, R, \delta \rangle$, where W is a set of worlds, and $@$ is a distinguished member of W. We may think of this as the actual world. R is the binary accessibility relation on W. No further constraints are imposed on R. (All the familiar ones are incorrect for exactly the same reasons as just discussed.) For every propositional parameter, p, $\delta(p)$ is a pair of sets of worlds, $\delta^+(p)$ and $\delta^-(p)$, such that $\delta^+(p) \cup \delta^-(p) = W$. Truth at a world, $\Vdash^+$, and falsity at a world, $\Vdash^-$, are then defined as follows:

- $w \Vdash^+ \neg A$ iff $w \Vdash^- A$

[15] See, e.g., Priest (2008), ch. 7.

- $w \Vdash^- \neg A$ iff $w \Vdash^+ A$
- $w \Vdash^+ A \wedge B$ iff $w \Vdash^+ A$ and $w \Vdash^+ B$
- $w \Vdash^- A \wedge B$ iff $w \Vdash^- A$ or $w \Vdash^- B$
- $w \Vdash^+ A \vee B$ iff $w \Vdash^+ A$ or $w \Vdash^+ B$
- $w \Vdash^- A \vee B$ iff $w \Vdash^- A$ and $w \Vdash^- B$

The truth conditions for the modal operators are, as expected:

- $w \Vdash^+ \Box A$ iff for all w' such that wRw', $w' \Vdash^+ A$
- $w \Vdash^+ \Diamond A$ iff for some w' such that wRw', $w' \Vdash^+ A$

An inference from the set of premises Σ to conclusion A is valid if for every interpretation, if $@ \Vdash^+ B$ for all $B \in \Sigma$, then $@ \Vdash^+ A$.

As is easy to check, it is now possible for there to be a world, w, where $w \Vdash^+ \Box A \wedge \Box \neg A$, even though for some B, $w \nVdash^+ \Box B$. (w accesses only worlds where both A and $\neg A$ hold.) Consequently, the inference $\Box A \wedge \Box \neg A \models \Box B$ is invalid—and *a fortiori*, so is Explosion. Note, however, that the semantics is not committed to dialetheism. There is no reason why we should have anything of the form $@ \Vdash^+ B \wedge \neg B$. All that is required is that there be *some* world, w, maybe an impossible world, where contradictory obligations are realised at w. The actual world does not have to be this world; that is, dialetheism does not have to be true.

Indeed, note that I have not given the falsity conditions for the modal operators. If we give these as:

- $w \Vdash^- \Box A$ iff $w \nVdash^+ A$
- $w \Vdash^- \Diamond A$ iff $w \nVdash^+ A$

and we require of every interpreation that for every propositional parameter, p, $@ \notin \delta^+(p) \cap \delta^-(p)$, then, as is easy to check, no contradictions hold at @. Indeed, the consequence relation restricted to non-modal formulas is exactly that of classical logic.

If one is to countenance dialetheism about norms then, clearly, this will not do. The easiest way to allow for such dialetheias is to give the falsity conditions of the modal operators in the usual way:

- $w \Vdash^- \Box A$ iff for some w such that wRw', $w' \Vdash^- A$
- $w \Vdash^- \Diamond A$ iff for all w such that wRw', $w' \Vdash^- A$

One may then have an interpretation where $@ \Vdash \Box A \wedge \neg\Box A$, and the logic is a modal extensions of LP.[16]

I note that the above semantics validate the "aggregation principle", $\Box A, \Box B \models \Box(A \wedge B)$, though they could be complicated not to do so. A prime motivation for denying this principle is the possibility of normative dilemmas themselves. If this inference is valid, then a dilemma delivers something of the form $\Box(A \wedge \neg A)$. Assuming that *ought implies can*, $\Box A \models \blacklozenge A$, it follows that $\blacklozenge(A \wedge \neg A)$; and it can hardly be the case that this is so for every dilemmatic A—even if it holds in occasional dialetheic cases. In fact, I take the *ought implies can* principle to be simply false.[17] Thus, for example, you may well promise to do something (say, repay a loan) with the reasonable expectation of being able to do so. However, it then turns out that you are not able to do so (perhaps because you lose your job). You then violate your obligation—though of course, there are extenuating circumstances.

Which brings us to $\blacksquare$ and $\blacklozenge$ themselves To handle these semantically, one needs a new binary accessibility relation, S, where:

- $w \Vdash^+ \blacksquare A$ iff for all w' such that wSw', $w' \Vdash^+ A$
- $w \Vdash^+ \blacklozenge A$ iff for some w' such that wSw', $w' \Vdash^+ A$

(We need not consider falsity conditions, since these play no role in what follows.) The inference $\Box A \models \blacklozenge A$ is obviously validated by the following condition connecting R and S:

- if for all w such that $@Rw$, $w \Vdash^+ A$, then for some w such that $@Sw$, $w \Vdash^+ A$

But as I have argued, that condition should not be imposed.

In §3, we met two characterisations of dilemmas:

[**1**] for some A, $\Box A \wedge \Box\neg A$

[**2**] for some A and B, $\Box A \wedge \Box B \wedge \blacksquare(A \to \neg B)$

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[16] More generally on modal many-valued logics, see Priest (2008), ch. 11a.

[17] See Priest (2006a), pp. 193-4.

To formalise the relation between these two, we need add an appropriate conditional, $\to$, to the language. The strict conditional corresponding to R will do nicely. Its truth conditons are:

- $w \Vdash^+ A \to B$ iff for all w' such that wRw', if $w \Vdash^+ A$ then $w \Vdash^+ B$.

(Again, we do not need to consider falsity conditions, since these play no role in what follows.)[18]

This delivers the inference from [1] to [2], as observed, since $\models \blacksquare(A \to \neg\neg A)$. For the inference in the other direction, we require, again as observed, the validity of the inference: $\blacksquare(A \to B), \Box A \models \Box B$. This inference is verified by the condition:

- $\forall x\, xSx$

(That is, inferentially, $\blacksquare A \models \blacklozenge A$. This is clearly correct: if you bring about A whatever you can do, then whatever you actually do—and so can do—will bring it about.) For then, suppose that $@ \Vdash^+ \blacksquare(A \to B)$. Then $@ \Vdash^+ A \to B$, and so for every w such that $@Rw$, if $w \Vdash^+ A$, $w \Vdash^+ B$. Now, suppose that $@ \Vdash^+ \Box A$. Then for every w such that $@Rw$, $\Vdash^+ A$. But then for every such w, $w \Vdash^+ B$. That is, $@ \Vdash^+ \Box B$.

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[1] Bolander, T. (2017), 'Self-Reference', in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/self-reference/.

[2] Chihara, C. (1979), 'The Semantic Paradoxes: a Diagnostic Investigation', *Philosophical Review* 88: 590-618.

[3] McConnell, T. (2018), 'Moral Dilemmas', in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/moral-dilemmas/.

[4] Priest, G. (1995), *Beyond the Limits of Thought*, Cambridge: Cambridge University Press; 2nd edn, Oxford: Oxford University Press, 2002.

[18] To make $\to$ satisfy *modus ponens*, one can add the clause 'or wRw' to the *definiens*. And to make $\to$ contrapose, one may add the clause: 'and for all w' such that wRw', if $w \Vdash^- B$ then $w \Vdash^- A$'.

[5] Priest, G. (2006a), *In Contradiction*, 2nd edn, Oxford: Oxford University Press.

[6] Priest, G. (2006b), *Doubt Truth to be a Liar*, Oxford: Oxford University Press.

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[8] Routley, R., and Plumwood, V. (1989), 'Moral Dilemmas and the Logic of Deontic Norms', pp. 653-690 of G. Priest, R. Routley, and J. Norman (eds.), *Paraconsistent Logic: Essays on the Inconsistent,* Munich: Philosophia Verlag.